



VECTOR (OR CROSS) PRODUCT OF TWO VECTORS



Properties of vector (or cross) product

Angle between two vectors

Lagrange's identity

if $\vec{a}, \vec{b}$ are any two vectors then $(\vec{a} \times \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$

Area of a parallelogram



Area of a triangle



Question 1.

(i) If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$, then find $|\vec{a} \times \vec{b}|$. (NCERT)

(ii) Find the magnitude of $\vec{a}$, where $\vec{a} = (\hat{i} + 3\hat{j} - 2\hat{k}) \times (-\hat{i} + 3\hat{k})$.

(iii) If $\vec{a} = 3\hat{i} - 5\hat{j}$, $\vec{b} = 6\hat{i} + 3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c} = \vec{a} \times \vec{b}$, then find $|\vec{a}| : |\vec{b}| : |\vec{c}|$.

Question 2.

- (i) Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively when $|\vec{a} \times \vec{b}| = \sqrt{3}$.
- (ii) If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3, |\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector then find the angle between $\vec{a}$ and $\vec{b}$.
- (iii) If $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

Question 3.

Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$. (NCERT)

Question 4.

(i) Find $\vec{a} \cdot \vec{b}$ if $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$.

(ii) If $|\vec{a}| = 4$; $|\vec{b}| = 2$ and angle between $|\vec{a}|$ and $|\vec{b}|$ is $\frac{\pi}{3}$, find $(\vec{a} \times \vec{b})^2$.

(iii) If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \frac{1}{2}$, $|\vec{b}| = \frac{4}{\sqrt{3}}$ and $|\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}}$ then find $|\vec{a} \cdot \vec{b}|$.

Question 5.



(i) Find the value of λ if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.

(ii) Find the value of p if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0}$.

(iii) Find λ and μ $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$.

(NCERT)

$$(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 14 \\ 1 & -\lambda & 7 \end{vmatrix} = \vec{0}$$

$$\vec{0} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\hat{i}(42 + 14\lambda) - \hat{j}(14 - 14) + \hat{k}(-2\lambda - 6) = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$42 + 14\lambda = 0$$

$$14\lambda = -42$$

$$\lambda = -42/14 = -3$$

$$-2\lambda - 6 = 0$$

$$\lambda = \frac{-6}{2} = -3$$

$$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0} \quad p = ?$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & 3 & p \end{vmatrix} = \vec{0}$$

$$\hat{i}(6p - 81) - \hat{j}(2p - 27) + \hat{k}(6 - 6) = \vec{0}$$

$$\hat{i}(6p - 81) - \hat{j}(2p - 27) + 0\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$6p - 81 = 0$$

$$6p = 81$$

$$p = \frac{81}{6} = \frac{27}{2}$$

$$2p - 27 = 0$$

$$2p = 27$$

$$p = \frac{27}{2}$$

$$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\hat{i}(6\mu - 27\lambda) - \hat{j}(2\mu - 27) + \hat{k}(2\lambda - 6) = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$6\mu - 27\lambda = 0$$

$$2\lambda - 6 = 0$$

$$2\mu - 27 = 0$$

$$2\lambda = 6$$

$$2\mu = 27$$

$$\lambda = 3$$

$$\mu = \frac{27}{2}$$

Question 6.

(i) Find a unit vector perpendicular to the two vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $2\hat{i} + 3\hat{j} + \hat{k}$. (ISC 2004)

(ii) Find a unit vector perpendicular to $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$$

$$|\hat{n}| = 1$$

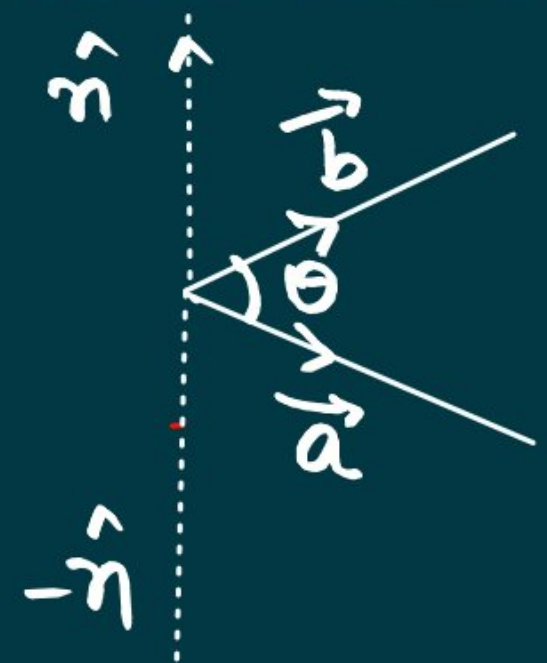
Let

$$\vec{a} = \hat{i} + 2\hat{j} - \hat{k} \quad \vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}(2+3) - \hat{j}(1+2) + \hat{k}(3-4)$$

$$\vec{a} \times \vec{b} = 5\hat{i} - 3\hat{j} - \hat{k}$$



$$|\vec{a} \times \vec{b}| = \sqrt{(5)^2 + (-3)^2 + (-1)^2}$$

$$= \sqrt{25 + 9 + 1}$$

$$= \sqrt{35}$$

$$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{5\hat{i} - 3\hat{j} - \hat{k}}{\sqrt{35}} \text{ Ans}$$

Question 7.

(i) What conclusion can you draw about vectors $\vec{a}$ and $\vec{b}$ when $\vec{a} \times \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 0$? (NCERT)

(ii) If either $\vec{a} = 0$ or $\vec{b} = 0$, then $\vec{a} \times \vec{b} = \vec{0}$. Is the converse true? Justify your answer with an example. (NCERT)

$$(i) \quad \vec{a} \times \vec{b} = 0, \quad \vec{a} = 0 \text{ or } \vec{b} = 0 \quad \vec{a} \parallel \vec{b} \text{ (Collinear)}$$

$$\vec{a} \cdot \vec{b} = 0, \quad \vec{a} = 0 \text{ or } \vec{b} = 0 \quad \vec{a} \perp \vec{b} \quad \theta = \frac{\pi}{2}$$

$$(ii) \quad \vec{a} = 0, \quad \vec{b} = 0 \quad \vec{a} \times \vec{b} = 0 \quad \text{True}$$

$$\vec{a} \times \vec{b} = 0 \quad \text{False.}$$

Question 8.

Find λ such that $\vec{a} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ are parallel.

Hint: $\vec{a} \times \vec{b} = \vec{0}$

$$\vec{a} = \hat{i} + \lambda\hat{j} + 3\hat{k} \quad \vec{b} = 3\hat{i} + 2\hat{j} + 9\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \lambda & 3 \\ 3 & 2 & 9 \end{vmatrix}$$

$$0 = \hat{i}(9\lambda - 6) - \hat{j}(9 - 9) + \hat{k}(2 - 3\lambda)$$

$$\hat{i}(9\lambda - 6) - 0 + \hat{k}(2 - 3\lambda) = 0$$

$$9\lambda - 6 = 0$$

$$\lambda = 6/9 = 2/3$$

Question 9.

(i) $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j}$

(ii) $(\hat{j} \times \hat{k}) \cdot \hat{i} + (\hat{i} \times \hat{k}) \cdot \hat{j} + (\hat{i} \times \hat{j}) \cdot \hat{k}$

Question 10.

Find the area of the parallelogram whose adjacent sides are represented by the vectors

(i) $3\hat{i} + \hat{j} + 4\hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ (NCERT)

(ii) $2\hat{i} - 3\hat{k}$ and $4\hat{j} + 2\hat{k}$